$\Rightarrow \left[\sum_{j\geq 1}^{\infty}|\xi_j|^p\right]^{1/p} < \infty$

$\Rightarrow x \in l^p$ where $x = \langle \xi_j \rangle_{j\geq 1}^{\infty} \in l^p$.

Also by ②, we have

$d(x_n, x) < \epsilon \quad \forall\, n \geq N.$

$\Rightarrow x_n \to x$ & $x \in l^p$.

So l^p space is Complete.

$\Rightarrow (l^p, \|\cdot\|_p)$ is Banach space.

(3) The sequence space l^∞ is Complete.

$$\|x\|_\infty = \sup_{1\leq j<\infty}|\xi_j| \quad \forall\, x = \langle \xi_j \rangle_{j\geq 1}^{\infty} \in l^\infty.$$

$$d_\infty(x,y) = \|x-y\|_\infty = \sup_{1\leq j<\infty}|\xi_j - \eta_j| \quad \forall\, x = \langle \xi_j \rangle_{j\geq 1}^{\infty},\ y = \langle \eta_j \rangle_{j\geq 1}^{\infty} \in l^\infty$$

Let $\langle x_n \rangle_{n\geq 1}^{\infty}$ be a Cauchy sequence in l^∞ s.t. $\{x_n\} = \langle \xi_j^n \rangle_{j\geq 1}$.

Then for each $\epsilon > 0$, $\exists$ a +ve integer N s.t.

$\|x_n - x_m\|_\infty < \epsilon \quad \forall\, n,m \geq N.$

$\Rightarrow d_\infty(x_n, x_m) < \epsilon \quad \forall\, n,m \geq N$

$\Rightarrow \sup_{1\leq j<\infty}|\xi_j^{(n)} - \xi_j^{(m)}| < \epsilon \quad \forall\, n,m \geq N.$

$\Rightarrow |\xi_j^{(n)} - \xi_j^{(m)}| < \epsilon \quad \forall\, n,m \geq N.$ —— ②

$\therefore$ for each fixed j, $\langle \xi_j^{(n)} \rangle_{n\geq 1}^{\infty}$ is a Cauchy seq. in $\mathbb{K}$ $(\underbrace{\mathbb{K} = \mathbb{R} \text{ or } \mathbb{C}}_{\text{both are complete}})$

$\Rightarrow \langle \xi_j^{(n)} \rangle_{n\geq 1}^{\infty}$ is Convergent in $\mathbb{R}$ $\forall\, j \in \mathbb{N}$

$\Rightarrow \lim_{n\to\infty} \xi_j^{(n)} = \xi_j \in \mathbb{K} \quad \forall\, j \in \mathbb{N}$

Using these infinite limit we define

$x = \langle \xi_1, \xi_2, \ldots \rangle$

Letting $m \to \infty$ in eq^n ②, we have,